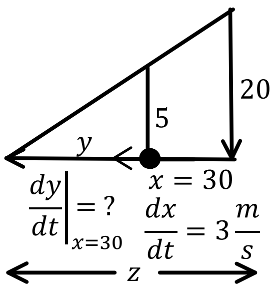
# C12 - 4.4 - Shapes/Similar Δ*/Eq Rel Rates Notes

A 5 foot tall woman walking away from a 20 foot lamp post at 3 m/s. Find rate is shadow increasing when 30 feet from post? How fast is the tip of her shadow moving away from post then?



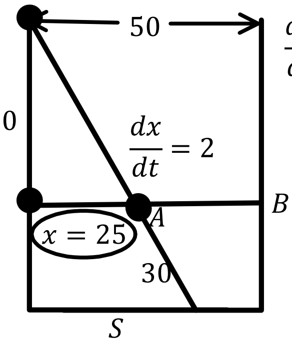

$\left.\frac{dy}{dt}\right|_{x=30} = ?$ $\quad x = 30 \quad \frac{dx}{dt} = 3\frac{m}{s}$

$\left.\frac{dz}{dt}\right|_{x=30} = ?$

$$\frac{5}{20} = \frac{y}{x+y}$$
$$5x + 5y = 20y$$
$$5x = 15y$$
$$x = 3y$$
$$\frac{dx}{dt} = 3\frac{dy}{dt}$$
$$3 = 3\frac{dy}{dt}$$
$$\frac{dy}{dt} = 1\frac{ft}{s}$$

$$\frac{dz}{dt} = \frac{dy}{dt} + \frac{dx}{dt}$$
$$\frac{dz}{dt} = 1 + 3$$
$$\frac{dz}{dt} = 4\frac{m}{s}$$

Tightrope-walker Shadow

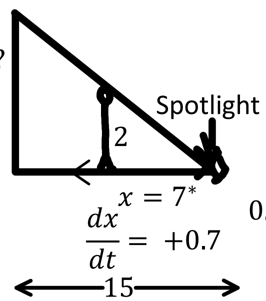



$\frac{dx}{dt} = 2$ $\quad x = 25$

$$\left.\frac{dS}{dt}\right|_{x=25} = ?$$
$$\frac{x}{70} = \frac{S}{100}$$
$$x = \frac{7}{10}S$$
$$\frac{dx}{dt} = \frac{7}{10}\frac{dS}{dt}$$
$$2 = \frac{7}{10}\frac{dS}{dt}$$
$$\frac{dS}{dt} = \frac{20}{7}\frac{m}{s}$$

A 2 m tall person is walking away from a spotlight, 15 m from a wall, towards the wall at 0.7 m/s. How fast is the shadow on the wall changing when they are 7 m from the spotlight?

$\left.\frac{dy}{dt}\right|_{x=7} = ?$



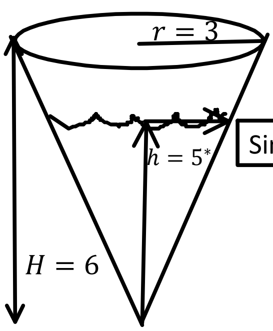

$x = 7^*$ $\quad \frac{dx}{dt} = +0.7$

$$\frac{y}{15} = \frac{2}{x}$$
$$xy = 30$$
$$\frac{dx}{dt}y + \frac{dy}{dt}x = 0$$
$$0.7(4.29) + \frac{dy}{dt}(7) = 0$$
$$\frac{dy}{dt} = -0.429\frac{m}{s}$$

*The shadow is decreasing at*

$$\frac{y}{15} = \frac{2}{x}$$
$$\frac{y}{15} = \frac{2}{7}$$
$$y = 4.29$$

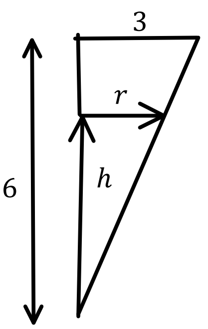
A cone of radius 3 cm & height 6 cm is filling with water where the height of the water level is increasing at a rate of 0.2 cm/s. Find the rate the volume is increasing when the height of the water level is 5 cm.




Similar Triangles

$\frac{dh}{dt} = 0.2 \quad \left.\frac{dV}{dt}\right|_{h=5} = ?$

$$\frac{H}{R} = \frac{h}{r}$$
$$\frac{6}{3} = \frac{h}{r}$$
$$r = \frac{h}{2}$$




$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$
$$V = \frac{1}{12}\pi h^3$$
$$\frac{dV}{dt} = 3 \times \frac{1}{12}\pi h^2 \frac{dh}{dt}$$
$$\frac{dV}{dt} = \frac{1}{4}\pi(5)^2(0.2)$$
$$\frac{dV}{dt} = \frac{5\pi}{4}\frac{cm}{s}$$

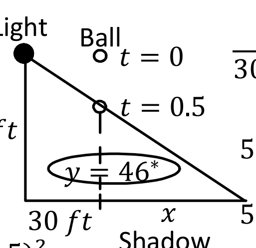
If it's draining into a cylinder use this #*

*We can't take this product so we must use similar triangles/other info $\quad \frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + \frac{dh}{dt}r^2\right)$

Ball Drop

$y = 50 - 16t^2$

$\left.\frac{dx}{dt}\right|_{t=0.5\ s} = ?$




$$y = 50 - 16t^2$$
$$y = 50 - 16(0.5)^2$$
$$y = 46\ ft$$

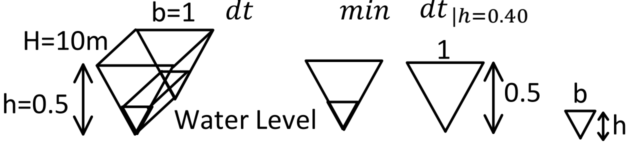
$$\frac{50}{30+x} = \frac{y}{x}$$
$$50x = 30y + xy$$
$$50\frac{dx}{dt} = 30\frac{dy}{dt} + \left(\frac{dx}{dt}y + \frac{dy}{dt}x\right)$$
$$50\frac{dx}{dt} = 30(-16) + \frac{dx}{dt}(46) + (-16)(345)$$
$$\frac{dx}{dt} = -1500\frac{ft}{s}$$

$$50x = 30y + xy$$
$$50x = 30(46) + 46x$$
$$4x = 1380$$
$$x = 345\ m$$

$$y = 50 - 16t^2$$
$$\frac{dy}{dt} = -32t$$
$$\frac{dy}{dt} = -32(0.5)$$
$$\frac{dy}{dt} = -16\frac{m}{s}$$

# C12 - 4.4 - Shapes/Similar Δ* Rel Rates Notes

## Water Filling




$\frac{dV}{dt} = 0.4\frac{m^3}{min}$ $\quad \frac{dh}{dt}\bigg|_{h=0.40} = ?$

$\frac{b}{1} = \frac{h}{0.5}$

$b = \frac{h}{\frac{1}{2}}$

$b = 2h$

$V = \frac{1}{2}bhH$

$V = \frac{1}{2}bh(10)$

$V = 5bh$

$V = 5(2h)h$

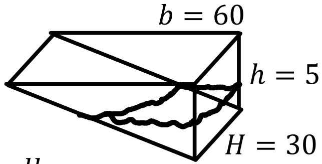
$V = 10h^2$

$\frac{dV}{dt} = 20h\frac{dh}{dt}$

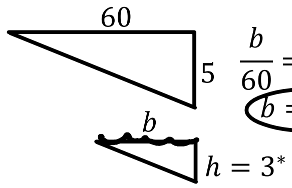
$0.4 = 20(0.4)\frac{dh}{dt}$

$\frac{dh}{dt} = 0.05\frac{m}{min}$

## Water Filling




$\frac{dh}{dt}\bigg|_{h=3} = ?$ $\quad \frac{dV}{dt} = 5\frac{m^3}{min}$




$\frac{b}{60} = \frac{h}{5}$

$b = 12h$

$V = \frac{bh}{2}H$

$V = \frac{bh}{2}(30)$

$V = 15bh$

$V = 15(12h)h$

$V = 180h^2$

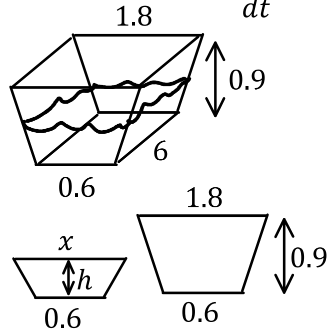
$\frac{DV}{dt} = 360h\frac{dh}{dt}$

$5 = 360(3)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{216}\frac{m}{min}$

## Water Filling

$\frac{dV}{dt} = 0.5$ $\quad \frac{dh}{dt}\bigg|_{h=0.5} = ?$




$\frac{x}{h} = \frac{1.8}{0.9}$

$x = 2h$

$V = \frac{a+b}{2} \times h \times H$

$V = \frac{x+0.6}{2} \times h \times 6$

$V = \frac{2h+0.6}{2} \times h \times 6$

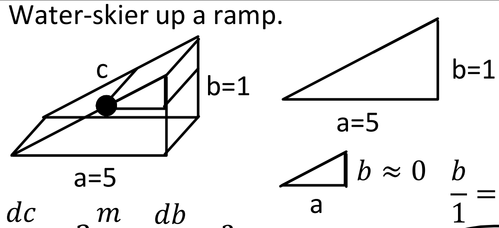
$V = 6h^2 + 1.8h$

$\frac{dV}{dy} = 12h\frac{dh}{dt} + 1.8\frac{dh}{dt}$

$0.5 = 12(0.5)\frac{dh}{dt} + 1.8\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{5}{76}$

## Water-skier up a ramp.




$\frac{dc}{dt} = 2\frac{m}{s}$ $\quad \frac{db}{dt}\bigg|_{b\approx 0} = ?$

$\frac{b}{1} = \frac{a}{5}$

$a = 5b$

$a^2+b^2 = c^2$

$(5b)^2+b^2 = c^2$

$26b^2 = c^2$

$52b\frac{db}{dt} = 2c\frac{dc}{dt}$

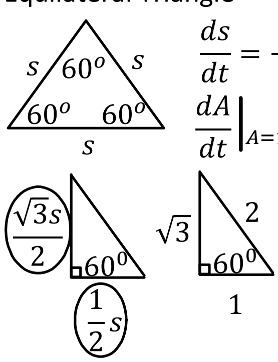
$52(1)\frac{db}{dt} = 2(\sqrt{26})(2)$

$a^2 + b^2 = c^2$

$c = \sqrt{26}$

$\frac{db}{dt} = \frac{2\sqrt{26}}{13}\frac{m}{s}$

## Equilateral Triangle




$\frac{ds}{dt} = -2\frac{cm}{s}$

$\frac{dA}{dt}\bigg|_{A=100} = ?$

$A = \frac{bh}{2}$

$A = \frac{1}{2}\left(s\frac{\sqrt{3}s}{2}\right)$

$A = \frac{\sqrt{3}s^2}{4}$

$A = \frac{\sqrt{3}s^2}{4}$

$\frac{DA}{dt} = \frac{1\sqrt{3}s}{2}\frac{ds}{dt}$

$\frac{DA}{dt} = \frac{1\sqrt{3}}{2}\left(\frac{20}{\sqrt[4]{3}}\right)(-2)$

$\frac{DA}{dt} = -20\sqrt[4]{3}\frac{cm^2}{s} = -26.3$

Wrong Text Answer

$-\frac{20}{\sqrt{3}} = -11.55$

$A = \frac{\sqrt{3}s^2}{4}$

$100 = \frac{\sqrt{3}s^2}{4}$

$\frac{400}{\sqrt{3}} = s^2$

$s = \frac{20}{\sqrt[4]{3}} = 15.2$

## Check Average*

$A = \frac{\sqrt{3}s^2}{4}$

$100 = \frac{\sqrt{3}s^2}{4}$

$s = 15.2$

| t | s |
|---|---|
| 0 | 15.2 |
| 1 | 13.2 |

$15.2 - 2$

$= 13.2$

$A = \frac{\sqrt{3}(13.2)^2}{4}$

$A = 75.5$

$\frac{dA^*}{dt} \approx \frac{DA}{ds} \times \frac{ds}{dt}$

$-24.5 \approx 12.25(-2)$ ✓

| s | A |
|---|---|
| 15.2 | 100 |
| 13.2 | 75.5 |

$\frac{A_2 - A_1}{s_2 - s_1}$

$\frac{75.5 - 100}{13.2 - 15.2}$

$\approx 12.25$

| t | A |
|---|---|
| 0 | 100 |
| 1 | 75.5 |

$\frac{A_2 - A_1}{t_2 - t_1}$

$\frac{75.5 - 100}{1 - 0}$

$\approx -24.5$